On the Infinite Past
or
Does the Universe have a Beginning?
or
Can History 'Count Down' from Negative Infinity?

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Abstract

To avoid the theological and philosophical implications of a beginning for the universe, some naturalists such as Sean Carroll suggest that all we need to do is build a successful mathematical model of the universe where time *t* runs from minus infinity to positive infinity. Although there is no problem in having *t* run from minus infinity to plus infinity with a mathematical model, the real past history of the universe cannot be a completed infinity of seconds that elapsed, one second at a time. There are at least two problems. First, an infinite real past requires a completed infinity, which is a single object and does not describe how history actually unfolds. Second, it is impossible to count down from negative infinity without encountering the problem of a potential infinity that never actually reaches infinity. For the real world, therefore, there must be a first event that occurred a finite amount of time ago in the past.

1. Introduction

As a result of our observations of the universe and its apparent ongoing expansion, the evidence from science points to a beginning. This raises the question of what might have caused physical reality to come into existence. Some atheists, understanding the possible theological implications, assert that physical reality, or nature, never had a beginning; the past is infinite either in terms of the number of seconds this universe has existed, or in terms of an infinite series of universes in a hypothetical multiverse that led up to this universe

Both of these ideas require an infinite number of real, sequential events in one form or another. As George Ellis and others have pointed out, the

multiverse is 'scientifically based philosophical speculation'.[1] This, of course, should not be confused with actual science. I will focus, therefore, on discussing the problem with an infinite past for the actual universe, though what will be argued here applies equally as well to an infinite series of past universes within a hypothetical multiverse.

Let us consider an argument made by Sean Carroll in his formal debate[2] with William Lane Craig. To avoid a beginning to the universe, Carroll suggested that all we have to do is to build a mathematical model of an eternal universe where 'time runs from minus infinity to plus infinity'. There is certainly no problem doing this with mathematical models, but to avoid a beginning for this actual universe, we must be able to transpose the mathematical model of an eternal universe to reality. In the real, physical world, could actual history have consisted of an infinite number of seconds that have literally run 'from minus infinity' down to the present?

It may not be patently obvious that history cannot count down from minus infinity to zero one integer at a time and arrive at this point. The article on potential and completed infinities referenced below makes statements such as "Infinity cannot be experienced in our everyday lives," and "If we don't see infinity in the physical world around us, then where do we see it? Why, in our heads, of course." One gathers from this that we must make a very clear distinction between mathematical concepts that exist only in our heads, and reality. There is often confusion between the two.

It is absolutely essential for this discussion that one have a clear understanding of the following terms as described in the embedded online links.

Countable infinity:[3] Any set which can be put in a one-to-one correspondence with the natural numbers or integers so that a prescription can be given for identifying its members one at a time. Example: the complete set of natural numbers.

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[1] George Ellis, *Scientific American*, 'Does the Multiverse Really Exist?'
[2] https://www.youtube.com/watch?v=X0qKZqPy9T8
[3] http://mathworld.wolfram.com/CountablyInfinite.html

Uncountable infinity:[4] An infinite set that is not countable. Example: the real numbers or the number of mathematical points in time during one second of elapsed time.

potential infinity:[5] refers to a procedure that gets closer and closer to, but never quite reaches, an infinite end. Examples: the sequence of numbers 1,2,3, … gets higher and higher but it has no end and it never gets to infinity; the limit of a function as *x* approaches infinity.

actual or completed infinity:[6] is an infinity that one actually reaches; the process is already done. It is just *one* object, a set. Example: the set of all positive integers {1,2,3, … }

Taking the example of all positive integers {1,2,3,4, ... } and quoting from the article above on the completed infinity,

> ... This is just *one* object, a set. But that set has infinitely many members. By that I don't mean that it has a large finite number of members and it keeps getting more members. Rather, I mean that it already *has* infinitely many members."

There are two key ideas to grasp. First, a completed countable infinity must be treated as a single 'object'. Secondly, it is impossible to count *to* a completed infinity, as mentioned in the definition of a potential infinity, one can count towards a potential infinity but never actually arrive at a completed infinity. Instead, for a completed infinity, 'the process is already done'. In short, one does not build a completed infinity, or worry about how each individual member got included; it is a single object that is delivered as a single, complete unit, not one member at a time.

It should also be noted that we are not concerned with *uncountable* infinities. Some, for example, have pointed out that there are an infinite number of points in time between, say, two seconds ago and three seconds ago. That is true, but that sort of infinity is an uncountable infinity and does nothing to avoid a beginning. In discussing the past, we

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[4] http://mathworld.wolfram.com/UncountablyInfinite.html
[5] http://www.math.vanderbilt.edu/~schectex/courses/thereals/potential.html
[6] http://www.math.vanderbilt.edu/~schectex/courses/thereals/potential.html

are speaking in terms of actual seconds that elapsed, one at a time, each of which can be assigned an integer value and, therefore, past history is countable and, if infinite, it must also be a completed infinity.

2. Requirements of an infinite past history

For Sean Carroll's proposed mathematical models of an eternal universe to map to the real world, the actual past history would have to be a completed countable infinity of seconds, each of which actually occurred, that ran 'from minus infinity' down to $t = 0$. This requires the following:

1. If the actual past has no beginning, and we fix $t = 0$ to a specific date, the number of seconds in the past is a *completed* countable infinity.
2. The number of elapsed seconds in the future is a *potential* infinity of seconds 1, 2, 3, … that have elapsed, or will elapse, but will never actually reach infinity.
3. Each second of past history can be assigned a negative integer value that extends backward in time (relative to an observer at $t = 0$).
4. Past history does not actually proceed backward in time, but proceeds forward toward $t = 0$. (e.g., { ... -4, -3, -2, -1, 0})
5. Given the above, for any elapsed second in the past, there is an negative integer value that represents its distance in the past from $t = 0$.

3.0 Problems

Given the definitions of various infinities provided in Section 1, there are at least two problems with Sean Carroll's proposal of real time t running from minus infinity to plus infinity.

3.1 A completed infinity is a violation of how history elapses

Recall that a completed infinity is a *single object* that is not achieved one step at a time but is *already* completed. It is a mistake to think one can count down through a completed infinity one member at a time and arrive at zero. One can count along a potential infinity one member at a time, but not a completed infinity; it is a single object that is already infinite. But past history did not elapse as a single object that was

already infinite. To better understand this problem, consider the following.

From (5) in Section 2 above, the size of past history is equal to the absolute value of the smallest negative integer value in past history. For example, if the total size of past history was thirteen seconds, then the smallest negative integer in the set of all seconds in the past would be negative 13, so the absolute value of that negative integer is 13, which equals the total size of this very short past history. With this preamble in mind, consider the following argument, referring back to (3) and (5) of Section 2:

Argument A:

A1. Given (3) and (5), if past history actually ran "from negative infinity" and since 'negative infinity' is infinitely distant from t = 0, then, necessarily, the absolute value of the smallest negative integer is infinite.

A2. Given (A1), if past history is infinite, then there are actual elapsed seconds that correspond to integers that have an infinite absolute values.

A2. All integers have a finite value; integers with an infinite value do not exist

A3. Therefore, past history cannot have run 'from negative infinity' to zero one elapsed second at a time.

The problem one encounters in Argument A arises as a result of how real history elapses as opposed to the mathematical variable we call *t*. Sticking only to mathematical models, we can the space from negative infinity to t = 0 as a single object, but real history actually elapses one second at a time, not as a single completed infinite mathematical object. A completed infinity is a single object that is *already* infinite.

To avoid the problem of integers that have an infinite absolute value illustrated above, we must treat history as a completed infinity, a single object that is *already* infinite. So, if we fix t = 0 to a particular date, say, today's date, then history actually looks like this,

{ …, -3, -2, -1}, 0, 1, 2, 3, …

where the portion in the brackets is a *completed* infinity denoted by the set of all negative integers and the future is a *potential* infinity. Recall that a potential infinity never reaches infinity; it is more of an indication of the direction the series of seconds is going into the future.

Problem:

Recall from the description of a completed infinity supplied earlier, that the completed infinity in the brackets above is a single object, a complete set of past seconds appearing instantaneously at a point in the past immediately prior to $t = 0$. But we have artifacts and ancient manuscripts that appear to be very old, that give us the impression that history was elapsing as a normal sequence of events, at least for many seconds into the past, not as a single object. Of course, we can solve this problem as well as avoid infinite negative integers by moving the transition at $t = 0$ back into the past prior to our oldest artifact but we still face two problems:

i. no matter how far back in the past we place the single object of the completed infinity, the portion of history described by a completed infinity simply does not instantly occur as an infinite number of simultaneous and instantaneous events.

ii. we cannot move the completed infinity of past history so far back in time that we get rid of it, for moving $t = 0$ back in time is a exercise involving a potential infinity where, recall, one never reaches infinity. For that reason, we cannot move $t = 0$ infinitely far back in time by passing through all the intervening seconds. The only way to do it is to move $t = 0$ infinitely far back in the past as a single quantum move but, by doing that, we instantly create a completed infinity, a single object, which is precisely what we were attempting to avoid.

Summary argument 3.1:

S1. History is composed of a countable series of events elapsing in a sequence, one after another.
S2. A past that contains a completed infinite number of countable

events must occur as a single object that is already infinite, not built up one event at a time.

S3. Therefore, the past cannot contain a completed infinite number of events.

S4. If the past cannot contain a completed infinity of countable events, then the past must be finite and there was a beginning.

S5. Therefore, past history is finite and there was a beginning of time.

Conclusion 3.1: The past cannot be composed of a completed infinity of countable events or seconds; it had a beginning.

3.2 Past history cannot count down from infinity

A common objection that is raised, with regard to the impossibility of counting down from infinity, is that one can actually do it if one counts for an infinitely long time. This objection makes two mistakes. The first is the failure to distinguish between a completed infinity which must be treated as a single mathematical object, and a potential infinity which we *can* count along one member at a time. The second mistake is the assumption that a potential infinity actually reaches infinity. Now to clarify …

Let us assume that the past was composed of an infinite number of seconds represented by the completed infinite set of all negative integers. For any set of integers, to count the number of members in the set, it matters not where we start counting, or in what direction we count, so long as we do not count the same integer more than once and we do not miss any. If real history is composed of an infinite number of seconds that elapsed one after another, then it must actually be possible to 'count through' the completed infinite set of negative integers one integer at a time. To test the idea that it is possible to count through the entire infinite set of all negative integers, let us see if we can do it. Recall, although history 'counted down' in the direction from negative infinity to $t = 0$, since we are counting the number of members in the set of all integers representing the seconds in past history, it matters not which direction we count, as explained above. So let us begin

-1, -2, -3, …

Notice that the list of negative integers we have actually counted through is a *potential* infinity. Recall from the definition of a potential infinity that infinity is never actually reached. To put it another way, it is impossible to count for an *actual* infinite amount of time, only for a *potentially* infinite amount of time. The set of all negative integers can *never* by counted through one integer at a time if each integer represents one second. History, therefore, could not have 'counted down' through the infinite set of all negative integer-numbered seconds.

Summary argument 3.2:

S1'. If each second in past history is represented by a negative integer, and past history is infinite, then it is possible to count the number of members in the infinite set of all negative integers, one integer at a time.
S2'. It is impossible to count the number of members in the infinite set of all negative integers one integer at a time.
S3'. Therefore, it is impossible for actual history to have 'counted down' through the infinite set of all negative integers one second at a time.

Conclusion 3.2: It is impossible for past history to have counted down from negative infinity. It has a beginning.

4.0 One final point

There are various mathematical models that might claim to show how past history can actually be a countable infinity but each one suffers from the same problems outlined in Sections 3.1 and 3.2.

For example, imagine that there is a relationship between the diameter of the universe and elapsed time that is described by a curve on the positive quadrant of the X-Y plane, with a vertical asymptote approaching the Y-axis representing time and a horizontal asymptote approaching the X-axis representing the radius of the universe. Also, imagine that we are at a point on the curve where $x = 3$ meters for the radius of the universe. As x approaches 0 in the past, y approaches infinity, but as we see from an example provided in the referenced description of a potential infinity, this is a potential infinity, not a

completed infinity. If we insist that there was a time when $x = 0$, then according to the model, there was a countably infinite number of seconds when $x = 0$, not just one second. If the radius of the universe is dependent upon time and the radius of the universe must start at 0, then there will be an infinite number of seconds that must be counted down through, one second at a time, before the curve diverges from 0 and the universe begins to expand. But since, as shown in Section 3.2, it is impossible to count down through an actual infinite number of seconds, one second at a time, no matter how old the universe is, it will never have a radius larger than 0 by that model.

One could respond by pointing out that the universe actually does have a radius larger than zero and, therefore, it must be infinitely old by that model, but that is merely assuming the conclusion that one is trying to prove. If the universe does have a radius larger than 0, then past history must be finite, by the curve provided in that model (the curve does not actually run from $x = 0$, but from some x larger than 0).

5.0 Conclusion

Past history cannot include a completed infinity for at least two reasons. First, a completed infinite must be treated as a single object that is *already* infinite, which is not how history elapses, one second at a time. Second, it is impossible to count down from negative infinity, for the process of 'counting' through an infinite number of members in a set involves a potential infinity, which never actually reaches infinity.